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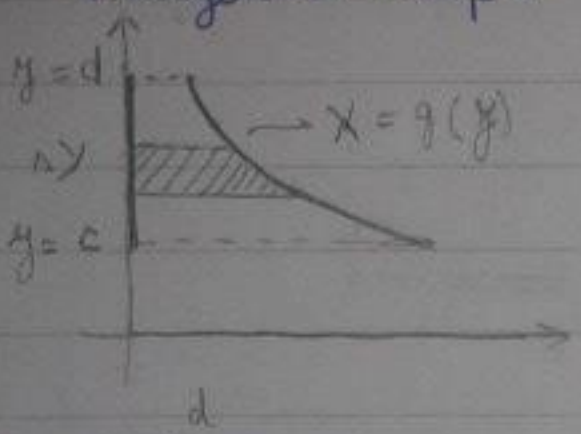
Application of Integration

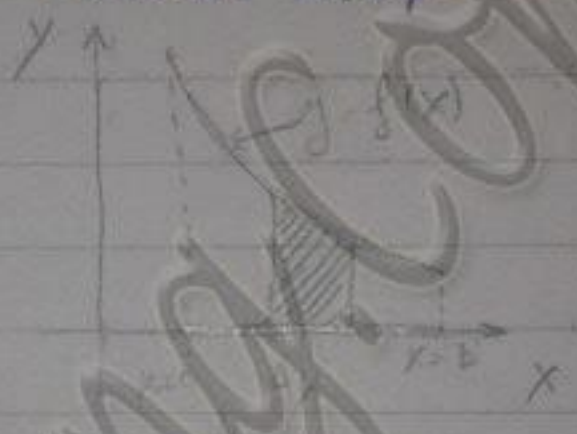
Computing Areas in Cartesian Coordinates

Horizontal Strips



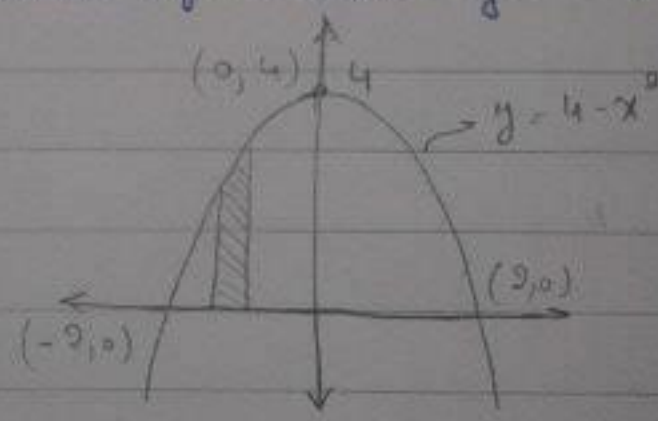
$$A = \int_c^d g(y)\,dy$$

Vertical Strips



Example 1-

Find the area of the region bounded by the curve $y = 4 - x^2$ and the x-axis.

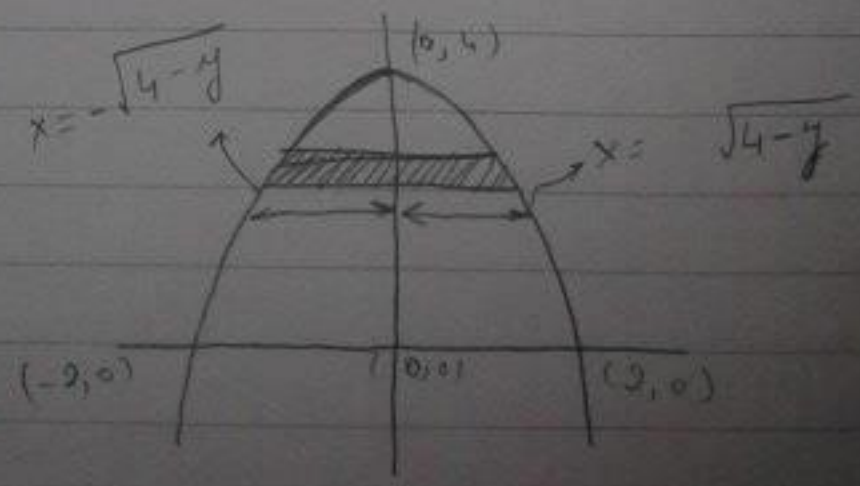


1) by using ver. strips:-

$$A = \int_{-2}^{2} (4 - x^2)\,dx = \frac{32}{3}$$

2) by Using Horizontal strips:-

$$A = 2\int_0^4 \sqrt{4-y}\,dy = \frac{32}{3}$$



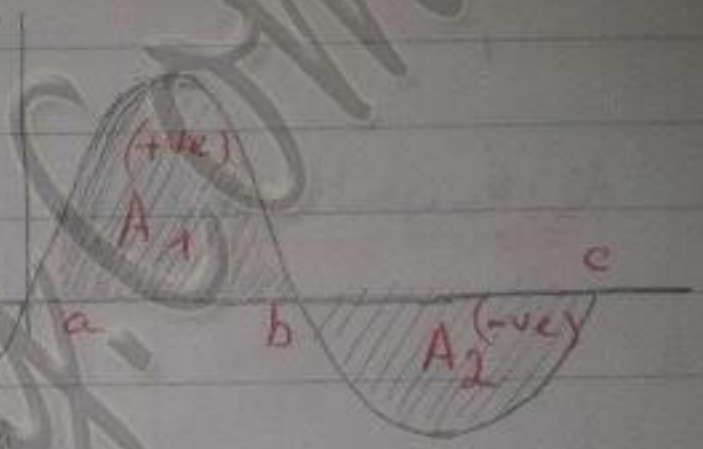
Remark:- (Sign of the area)

a) If $f(x) \geq 0$ over $[a,b] \Rightarrow A_1 = \int_a^b f(x)\,dx \geq 0$

b) If $f(x) \leq 0$ over $[b,c] \Rightarrow A_2 = \int_b^c f(x)\,dx \leq 0$

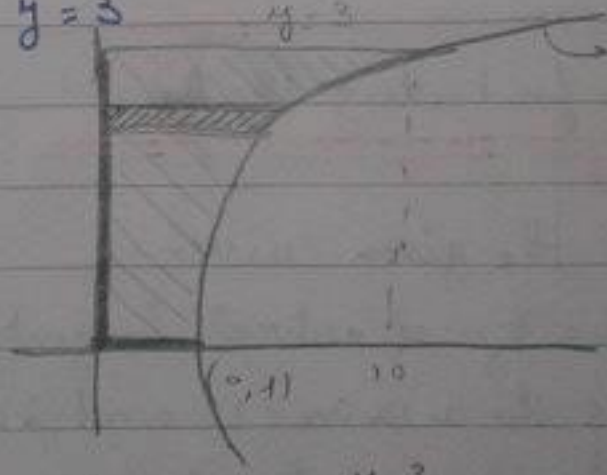
1) This implies that, the total Area A bounded by $y = f(x)$ over the interval $[a,b] \cup [b,c]$

$$A = |A_1| + |A_2|$$



2) to find the integration of the function on $y = f(x)$

Find the area of the region bounded by the curve $y^2 = x - 1$, the y-axis, the x-axis and the line $y = 3$

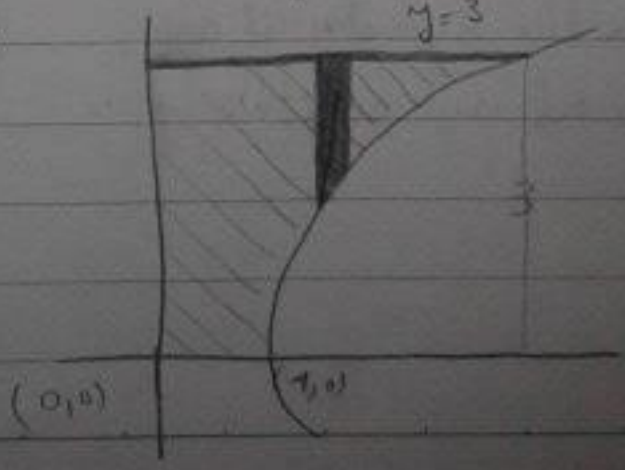


1) Horizontal:-

$$A = \int_0^3 x\,dy$$

2) Vertical,

$$A = \int_0^1 3\,dx + \int_1^{10} 3 - \sqrt{x-1}\,dx$$



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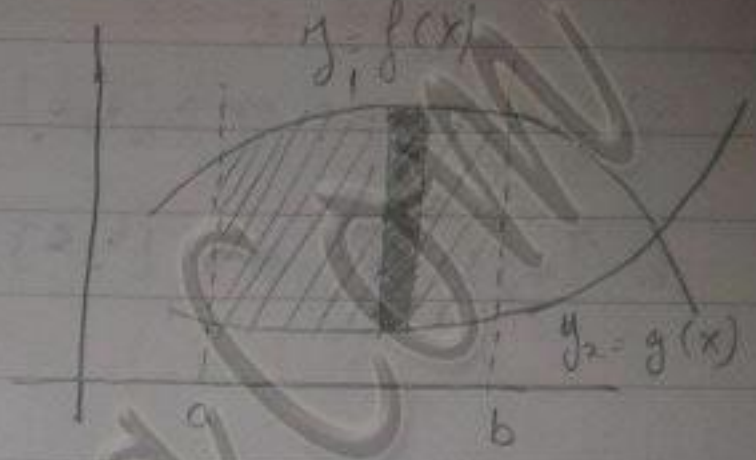
(6.1) Areas between Curves :-

A1) Vertical Strips :-

$$A = \int_a^b \left[f(x) - g(x) \right] dx$$

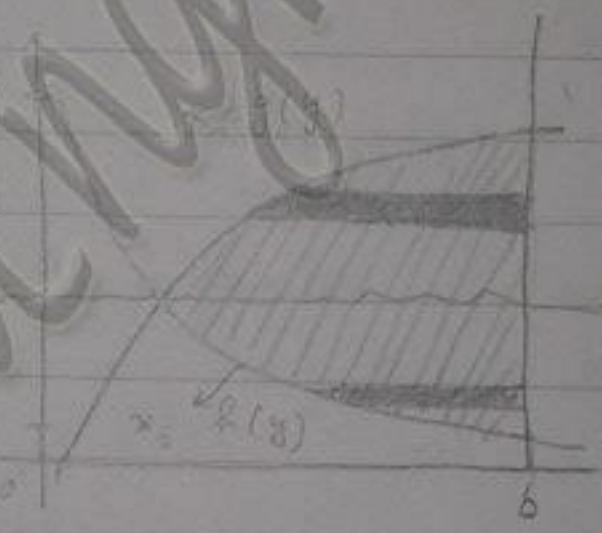


A2) Horizontal Strips :-

$$A_1 = \int_c^K (b - h(y))\, dy$$

$$A_2 = \int_K^d (b - g(y))\, dy$$

$$A = A_1 + A_2$$



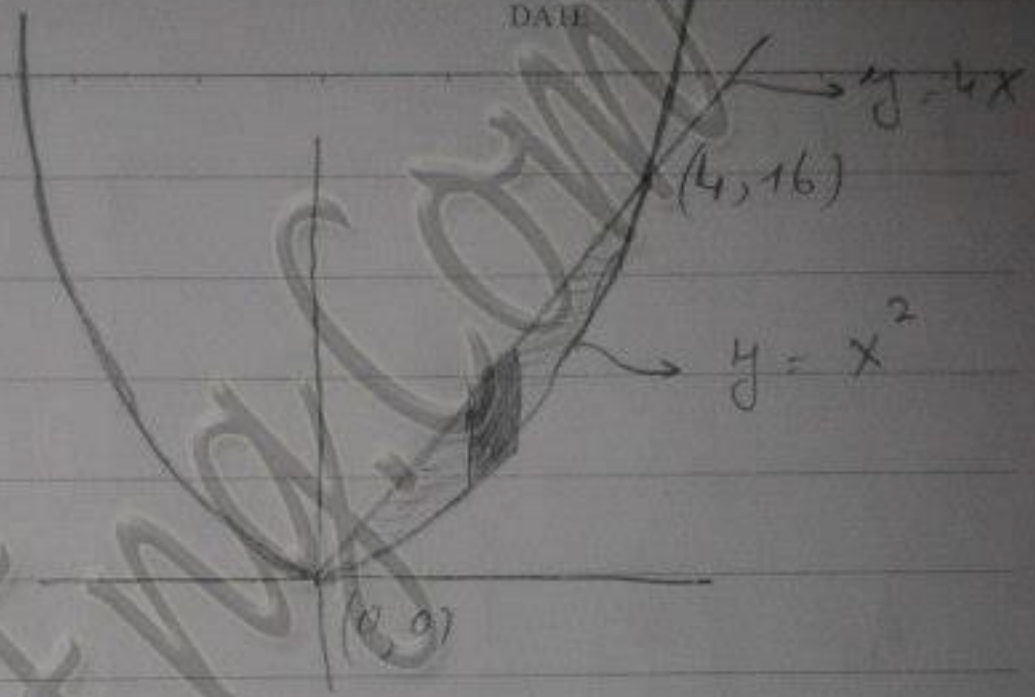
* It can be solved vertically

Guidelines for Finding the Areas :-

1) Sketch the given curve.
2) Sketch a typical vertical (horizontal) rectangle (strip)
3) Express the area in (1) and (2)
4) Consider the symetry if any.
5) Area is always positive (+)ve

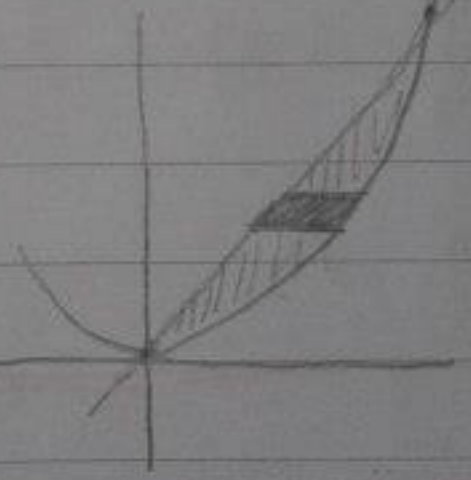
(1) Vertical Strips

$$A = \int_0^4 (4x - x^2)\,dx$$

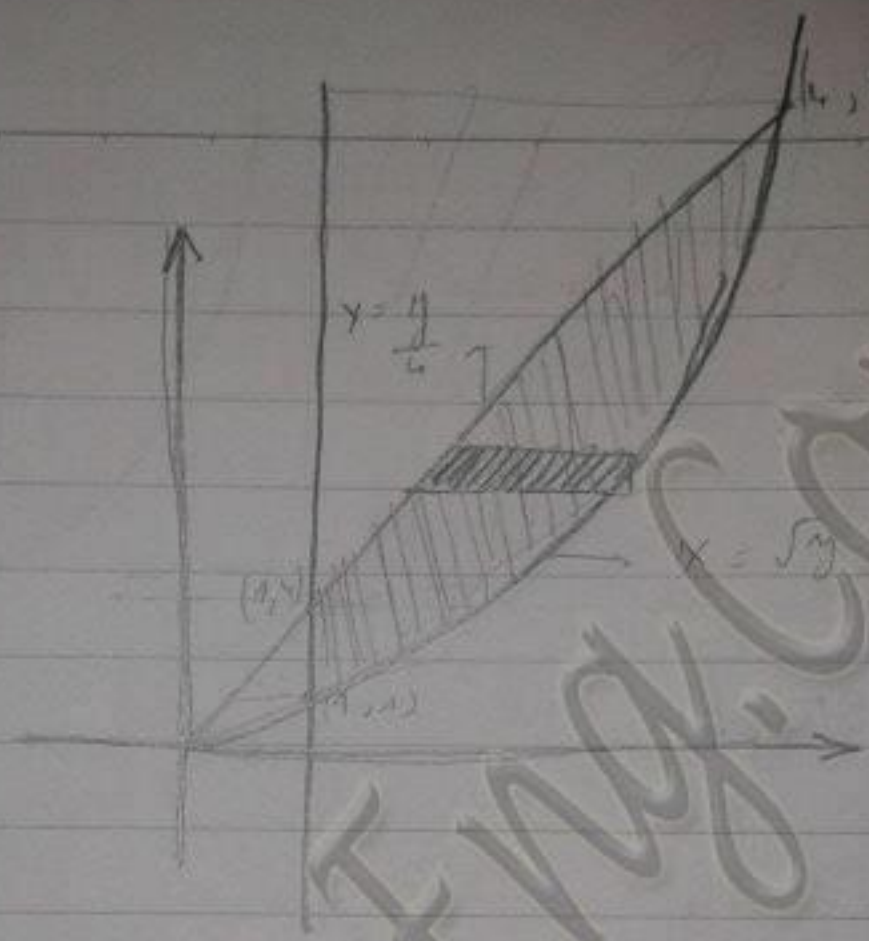


(2) Horizontal Strips :-

$$A = \int_0^{16} \left[\sqrt{y} - \frac{y}{4}\right] dy$$



Find the area:-



$$A_1 = \int_1^{16} \sqrt{y} - 1 \, dy$$

$$A_2 = \int_1^{4} \frac{y}{4} - 1 \, dy$$

$$A = A_1 - A_2$$

Example :-

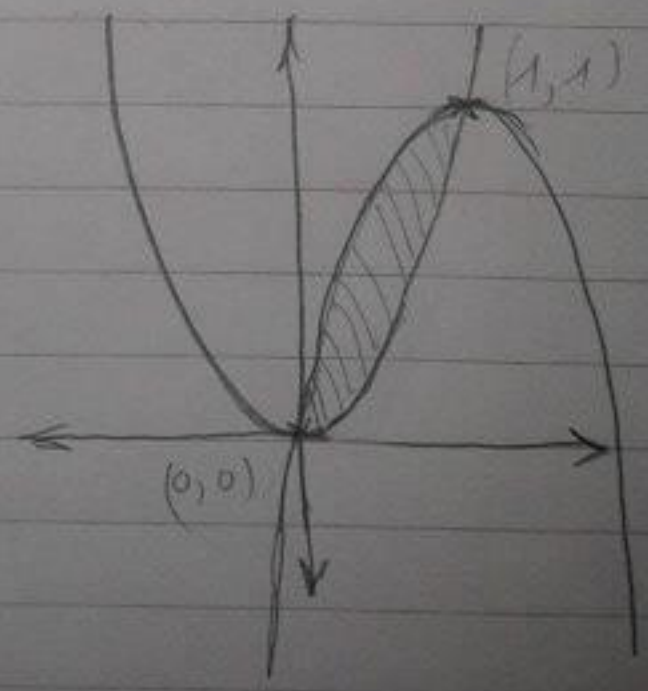
Find the area of the region bounded by the curve:-

1) $y = x^2$ & $y = 2x - x^2$

$-y = x^2 - 2x$

$-y = (x-1)^2 - 1$

$(y-1) = -(x-1)^2$



$$A = \int_0^1 [(2x - x^2) - (x^2)] \, dx = \frac{1}{3}$$

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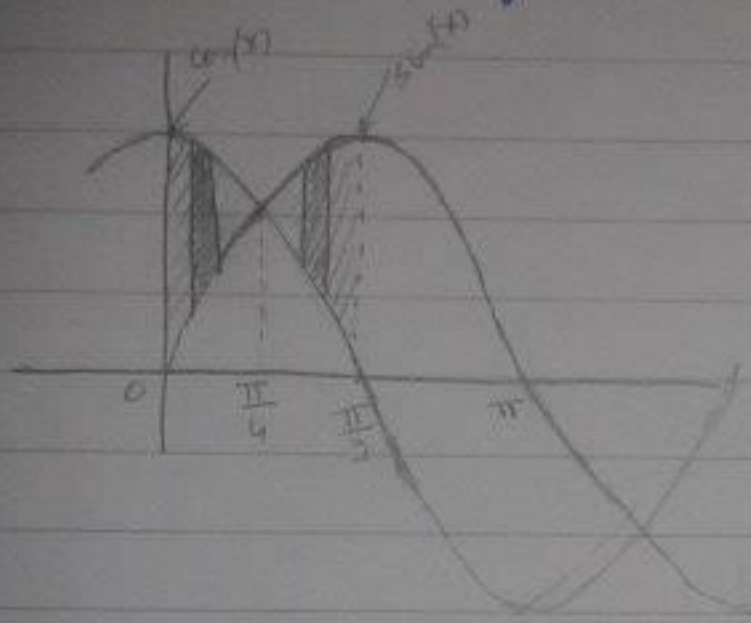
2) $y = \sin(x)$ & $y = \cos(x)$ $\quad x=0 \to x=\frac{\pi}{2}$



$$A_1 = \int_0^{\frac{\pi}{4}} \left[\cos(x) - \sin(x)\right] dx$$

$$A_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\sin(x) - \cos(x)\right] dx$$

$\Rightarrow |A_1| = |A_2|$

$A = \sqrt{2}$

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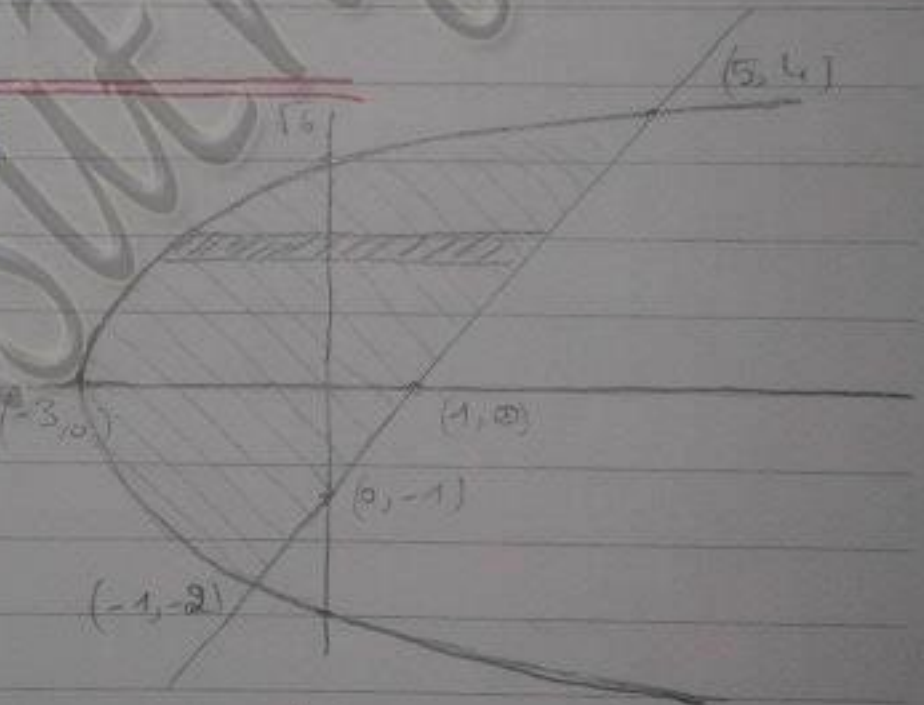
3) $y = x-1$ & $y^2 = 2x+6$

Horiz

$$A = \int_{-2}^{4} \left[(y+1) - \frac{1}{2}(y^2-6)\right] dy$$



Vertical

$$A_2 = \int_{-1}^{5} \left[\sqrt{2x+6} - (x-1)\right] dx$$

$$A_1 = 2\int_{-3}^{-1} \left[\sqrt{2x-6} - 0\right] dx$$

Sym about $y=0$